

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 1. (Currently amended) A method for using interval techniques
2 within a computer system to solve a multi-objective optimization problem,
3 comprising:
4 receiving a representation of multiple objective functions (f_1, \dots, f_n) at
5 the computer system, wherein (f_1, \dots, f_n) are scalar functions of a vector
6 $\mathbf{x} = (x_1, \dots, x_n)$;
7 receiving a representation of a domain of interest for the multiple
8 objective functions;
9 storing the representations in a memory within the computer system; and
10 performing an interval optimization process to compute guaranteed
11 bounds on a Pareto front for the objective functions (f_1, \dots, f_n), wherein for
12 each point on the Pareto front, an improvement in one objective function cannot
13 be made without adversely affecting at least one other objective function;
14 wherein performing the interval optimization process involves applying a
15 direct-comparison technique between subdomains of the domain of interest to
16 eliminate subdomains that are certainly dominated by other subdomains,
17 wherein performing the interval optimization process involves applying a
18 gradient technique to eliminate subdomains that do not contain a local Pareto
19 optimum,

20 wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an
21 intersection of certainly negative gradient regions \mathbf{C}_j for each objective function f_j
22 is non-empty, $\bigcap_{j=1}^n \mathbf{C}_j([\mathbf{x}]_i) \neq \emptyset$, and
23 wherein the certainly negative gradient region \mathbf{C}_j for objective function f_j
24 is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the
25 minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the
26 negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of
27 f_j over the subdomain $[\mathbf{x}]_i$).

1 2. (Cancelled)

1 3. (Cancelled)

1 4. (Currently amended) The method of ~~claim 3~~ claim 1, wherein the
2 method further comprises iteratively:
3 bisecting remaining subdomains that have not been eliminated by the
4 gradient technique; and
5 applying the gradient technique to eliminate bisected subdomains that do
6 not contain a local Pareto optimum.

1 5. (Original) The method of claim 4, wherein bisecting a subdomain
2 involves bisecting the subdomain in the direction that has the largest width of
3 partial derivatives of all objective functions (f_1, \dots, f_n) over the subdomain.

1 6. (Original) The method of claim 4, wherein the direct-comparison
2 technique is applied once for every n iterations of the gradient technique.

1 7. (Original) The method of claim 6, wherein the iterations continue
2 until either a predetermined maximum number of iterations are performed, or the
3 largest area of any subdomain is below a predetermined value.

1 8. (Original) The method of claim 1,
2 wherein a subdomain **U** certainly dominates a subdomain **V** if every point
3 **u** \in **U** dominates every point **v** \in **V**; and

4 wherein a point **u** dominates a point **v** under minimization if,

5 $u_i \neq v_i, i = 1, \dots, n$, and

6 $u_i < v_i$ for some $i \in \{1, \dots, n\}$.

1 9. (Currently amended) A computer-readable storage medium storing
2 instructions that when executed by a computer cause the computer to perform a
3 method for using interval techniques within a computer system to solve a multi-
4 objective optimization problem, wherein the computer-readable storage medium
5 can be any device that can store code and/or data for use by a computer system,
6 the method comprising:

7 receiving a representation of multiple objective functions (f_1, \dots, f_n) at
8 the computer system, wherein (f_1, \dots, f_n) are scalar functions of a vector
9 $\mathbf{x} = (x_1, \dots, x_n)$;

10 receiving a representation of a domain of interest for the multiple
11 objective functions;

12 storing the representations in a memory within the computer system; and

13 performing an interval optimization process to compute guaranteed

14 bounds on a Pareto front for the objective functions (f_1, \dots, f_n), wherein for

15 each point on the Pareto front, an improvement in one objective function cannot
16 be made without adversely affecting at least one other objective function;

17 wherein performing the interval optimization process involves applying a
18 direct-comparison technique between subdomains of the domain of interest to
19 eliminate subdomains that are certainly dominated by other subdomains,
20 wherein performing the interval optimization process involves applying a
21 gradient technique to eliminate subdomains that do not contain a local Pareto
22 optimum,
23 wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an
24 intersection of certainly negative gradient regions \mathbf{C}_j for each objective function f_j
25 is non-empty, $\bigcap_{j=1}^n \mathbf{C}_j([\mathbf{x}]_i) \neq \emptyset$, and
26 wherein the certainly negative gradient region \mathbf{C}_j for objective function f_j
27 is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the
28 minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the
29 negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of
30 f_j over the subdomain $[\mathbf{x}]_i$).

1 10. (Cancelled)

1 11. (Cancelled)

1 12. (Currently amended) The computer-readable storage medium of
2 ~~claim 11~~ claim 9, wherein the method further comprises iteratively:
3 bisecting remaining subdomains that have not been eliminated by the
4 gradient technique; and
5 applying the gradient technique to eliminate bisected subdomains that do
6 not contain a local Pareto optimum.

1 13. (Original) The computer-readable storage medium of claim 12,
2 wherein bisecting a subdomain involves bisecting the subdomain in the direction
3 that has the largest width of partial derivatives of all objective functions (f_1 , ..., f_n) over the subdomain.
4

1 14. (Original) The computer-readable storage medium of claim 12,
2 wherein the direct-comparison technique is applied once for every n iterations of
3 the gradient technique.
4

1 15. (Original) The computer-readable storage medium of claim 14,
2 wherein the iterations continue until either a predetermined maximum number of
3 iterations are performed, or the largest area of any subdomain is below a
4 predetermined value.

1 16. (Original) The computer-readable storage medium of claim 9,
2 wherein a subdomain **U** certainly dominates a subdomain **V** if every point
3 u \in **U** dominates every point **v** \in **V**; and
4 wherein a point **u** dominates a point **v** under minimization if,
5 $u_i \neq v_i, i = 1, \dots, n$, and
6 $u_i < v_i$ for some $i \in \{1, \dots, n\}$.

1 17. (Currently amended) An apparatus that uses interval techniques to
2 solve a multi-objective optimization problem, comprising:
3 a receiving mechanism configured to receive a representation of multiple
4 objective functions (f_1 , ..., f_n), wherein (f_1 , ..., f_n) are scalar functions of a
5 vector **x** = (x_1 , ..., x_n);
6 wherein the receiving mechanism is configured to receive a representation
7 of a domain of interest for the multiple objective functions;

8 a memory configured to store the representations; and
 9 an interval optimizer configured to performing an interval optimization
 10 process to compute guaranteed bounds on a Pareto front for the objective
 11 functions (f_1, \dots, f_n) , wherein for each point on the Pareto front, an
 12 improvement in one objective function cannot be made without adversely
 13 affecting at least one other objective function;
 14 wherein the interval optimizer is configured to apply a direct-comparison
 15 technique between subdomains of the domain of interest to eliminate subdomains
 16 that are certainly dominated by other subdomains,
 17 wherein the interval optimizer is configured to apply a gradient technique
 18 to eliminate subdomains that do not contain a local Pareto optimum,
 19 wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an
 20 intersection of certainly negative gradient regions \mathbf{C}_j for each objective function f_j
 21 is non-empty, $\bigcap_{j=1}^n \mathbf{C}_j([\mathbf{x}]_i) \neq \emptyset$, and
 22 wherein the certainly negative gradient region \mathbf{C}_j for objective function f_j
 23 is the intersection of $\mathbf{N}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the
 24 minimum angle θ_j of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the
 25 negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of
 26 f_j over the subdomain $[\mathbf{x}]_i$).

1 18. (Cancelled)

1 19. (Cancelled)

1 20. (Currently amended) The apparatus of ~~claim 19~~ claim 17, wherein
 2 the interval optimizer is configured to iteratively:

3 bisect remaining subdomains that have not been eliminated by the gradient
4 technique; and to
5 apply the gradient technique to eliminate bisected subdomains that do not
6 contain a local Pareto optimum.

1 21. (Original) The apparatus of claim 20, wherein bisecting a
2 subdomain involves bisecting the subdomain in the direction that has the largest
3 width of partial derivatives of all objective functions (f_1, \dots, f_n) over the
4 subdomain.

1 22. (Original) The apparatus of claim 20, wherein the direct-
2 comparison technique is applied once for every n iterations of the gradient
3 technique.

1 23. (Original) The apparatus of claim 22, wherein the iterations
2 continue until either a predetermined maximum number of iterations are
3 performed, or the largest area of any subdomain is below a predetermined value.

1 24. (Original) The apparatus of claim 17,
2 wherein a subdomain \mathbf{U} certainly dominates a subdomain \mathbf{V} if every point
3 $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and
4 wherein a point \mathbf{u} dominates a point \mathbf{v} under minimization if,
5 $u_i \neq v_i, i = 1, \dots, n$, and
6 $u_i < v_i$ for some $i \in \{1, \dots, n\}$.